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Mixing with Helical Ribbon Agitators

Of necessity, the mixing process is sometimes restricted to the laminar regime, although turbulent mixing is generally more desirable. Common examples of laminar mixing are found when the fluid has a very high viscosity, or when one of the mixture components is shear sensitive. It has been pointed out that the helical ribbon agitator (HRA) is admirably suited to the low Reynolds number mixing process. This work derives a model to predict the power consumption of the HRA. The model has been developed with the aid of experimental data and tested extensively using literature data. For a wide range of mixer geometries and sizes, it predicts power consumption with an average deviation of 13%.

The concept of relative efficiency of mixers is also described as an aid to comparing different HRA mixer geometries. Finally, the problem of scale-up of different HRA configurations is discussed.

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Part II. Newtonian Fluids

SCOPE

The mixing of very viscous fluids is qualitatively and quantitatively quite different from the process of blending low viscosity fluids. Parker (1964) noted that the helical ribbon agitator (HRA) is particularly well suited for mixing viscous liquids where the mixer flow is laminar. This observation is supported by the data of Gray (1963) and Johnson (1967). The choice of a mixer configuration is governed by three characteristic parameters: mixing time, power consumption, and mixer efficiency. The efficiency of a mixer is related to the total energy required to achieve a given degree of homogeneity. Mixing efficiency has been virtually ignored by previous workers, undoubtedly due to the impossibility of finding a usable definition of the (thermodynamically) perfect mixing process. This difficulty can be avoided by defining an efficiency relative to an arbitrary, standard mixer. This work proposes a standard mixer. This work proposes a standard mixer, and the relative efficiency $(\mathrm{eff})_{\mathrm{rel}}$ has been used to evaluate five HRA's of different geometries.

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Models for predicting the power consumption of HRA's have been proposed by Nagata et al. (1957), Bourne and Butler (1965, 1969), Chavan and Ulbrecht (1973*b*), and Hoogendoorn and Den Hartog (1967). [The state of mixing science has been exhaustively examined in Nagata's (1975) monograph.] These models are not generalized in the sense that one or more geometrical or fluid parameters have been neglected. This is not surprising, since dimensional analysis shows that ten dimensionless groups must be considered in the complete analysis. This paper proposes a general model developed from a drag flow analysis about the blades, using the observed hydrodynamics of the fluid flow during mixing. Most practical HRA's have certain dimensionless groups that vary only within narrow limits; thus, for engineering purposes, it is possible to obtain a simplified model employing only three dimensionless groups. This model has been verified with eight different geometries and three fluids. A slightly altered model was checked for fourteen different mixing conditions obtained from literature.

Finally, since almost all work done on HRA mixers has been on a laboratory scale (including this work), the question of scale-up is briefly examined.

CONCLUSIONS AND SIGNIFICANCE

The problem of defining and measuring the absolute efficiency of the mixing process has been avoided. A relative efficiency based on an arbitrary reference mixer was defined and has been found useful for comparing the performance of different mixers. In this way, it was found that the impeller efficiency increased as the blade width increased and the blade-vessel wall gap increased. A single-blade impeller was found to be significantly (~2.5 times) more efficient than the twin-blade agitator of similar geometry.

A model for the power consumption of the helical ribbon agitator was developed from drag flow analysis and the observed hydrodynamics of the flow. The original model could be drastically simplified by taking average values of some of the parameters that have little effect on the power consumption. The resulting model gave excellent agreement for the eight different mixer geometries and the three fluids examined. This model was slightly modified so as to be comparable to results reported in the literature; then the average deviation between experimental and predicted values was found to be 13% for fourteen different geometries. Part of this deviation may be ascribed to error in the approximate calculation of the blade length to diameter ratio l/d not reported by the previous workers.

The most important parameters affecting the power consumption were found to be the diameter ratio of the agitator to vessel d/D, the blade length to diameter ratio l/d, and the agitator pitch. Power consumption increased as the gap between the agitator and vessel wall decreased (increased d/D) and as the blade length increased (increasing l/d). Conversely, increasing the pitch ratio of the agitator decreased the power consumed, but this was at the expense of the mixing efficiency. Interestingly, the power consumption was not strongly affected by the blade width. However, the pumping capacity of the wide bladed agitator was markedly superior, giving this impeller a high efficiency.

The data and results reported here apply to a left-handed helical agitator rotated anticlockwise; that is, the impeller pumping action was downwards at the vessel wall. A few data were taken with the same impellers rotated in the opposite direction, pumping upwards at wall, and it was found that the power consumed was approximately 10% greater for this condition. This effect is not accounted for in the proposed model which was characterized for the downwards pumping flow only.

Mixing is a process that is basic to many industries and has been the subject of much investigation. Holland and Chapman (1966), Oldshue (1966), Rushton (1954), and Voncken (1965) have each compiled comprehensive reviews of fluid mixing, and Bourne (1964) has reviewed the mixing of powders, pastes, and non-Newtonian fluids. With the exception of Bourne's review, most of the publications that have appeared are concerned with the mixing of Newtonian fluids and have primarily discussed turbulent mixing. This is not surprising, since turbulent mixing is a much quicker process than mixing done in the laminar regime. Nonetheless, there are situations where mixing is restricted to the laminar regime. Examples of this situation include cases where the fluid has a very high viscosity, such as is possessed by polymer solutions or melts, or where one or more of the mixture components is shear sensitive. These cases are characterized by a low Reynolds number during mixing, where mixing proceeds through fluid deformation and not by turbulent eddy diffusion.

Parker (1964) in an article on modern practice on mixing, reported the recent introduction of the helical ribbon agitator which was effective in overcoming the large viscous forces present in the mixing of high viscosity fluids. The pertinent geometry of this type of agitator is shown in Figure 1.

As shown by many authors (Coyle et al., 1970; Gray, 1963; Hoogendoorn and Den Hartog, 1967; Moo-Young et al., 1972; Nagata et al., 1957, 1971), the time necessary to achieve a certain degree of homogeneity (mixing time) in viscous fluids is much shorter with a helical ribbon impeller than with a turbine (most commonly used) or with other impellers. The helical ribbon agitator and the helical screw draught tube system (Chavan and Ulbrecht, 1973*A*) are probably the most efficient mixers in the laminar regime. The inclined surface of the ribbon moving near the vessel wall produces axial movement of the liquid as well as tangential flow (Gray, 1963). Thus, a relatively effective top to bottom flow motion is obtained, compared with other impeller types.

Although the power requirement of the helical ribbon mixer is much higher than that of turbines or propellers at the same Reynolds number (Gray, 1963; Hoogendoorn and Den Hartog, 1967; Johnson, 1967), the energy required to obtain a given degree of homogeneity is considerably less. Gray (1963) compared mixing and power data for various impellers and concluded that the helical ribbon agitator mixes the most rapidly and requires the

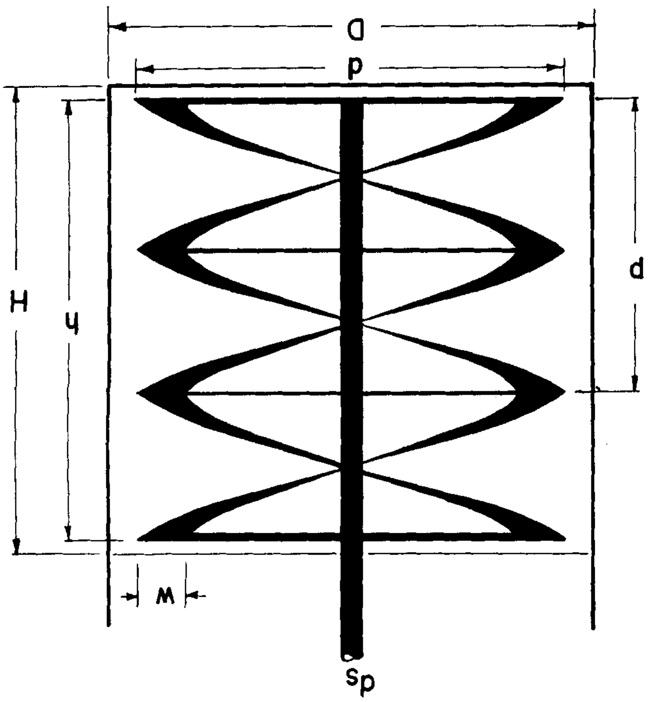

Fig. 1. Sketch of helical ribbon agitator system.

least energy. This was confirmed by Johnson (1967) who also showed that the helical ribbon mixers have a definite advantage for very viscous fluids. His data, obtained with corn syrup of 2 500 centipoises, indicated that at the same mixing effectiveness (mixing time of 5 min), the power required for the helical ribbon agitator was one sixth to one twelfth of the power required for turbines and propellers.

The mixing time requirement has been determined for a number of helical ribbon agitator geometries (Carreau et al., 1976), and this presentation is restricted to the development of a model to predict the power consumption of helical ribbon agitators mixing Newtonian fluids. The model has been tested using values reported in the literature in addition to data measured by us. A short discussion on the relative efficiencies of different helical ribbon agitators concludes the presentation.

THEORETICAL CONSIDERATIONS

Although we are primarily concerned with the prediction of power consumption, the test of a mixer's utility is its efficiency; that is, how much energy must be expended to obtain a specified degree of homogeneity. Thus, we digress temporarily to consider mixing time and relative mixing efficiency.

Mixing Time

The mixing time t_m is usually defined as the time required to reach a specified level of uniformity in a mixing system. It is a quantitative measurement of the time necessary to reduce the intensity of segregation (Danckwerts, 1953; Lacey, 1954) to a specified value which is usually taken in the order of 0.1 to 1%.

For nonturbulent mixing, however, the intensity of segregation is difficult to measure because of the difficulties involved in sampling. Thus, various other criteria for mixing time have been developed. The methods for measuring mixing time include electrical conductivity changes, refractive index differences, color differences, indicator changes during an ionic reaction, and temperature differences (Ford et al., 1972; Voncken, 1965; Hoogendoorn and Den Hartog, 1967). Nevertheless, because of the different criteria used with different methods, the effects of additives, and differing sampling philosophies, the mixing times obtained cannot be taken as absolute values. They are useful primarily as relative values, defined for a particular system and experimental technique.

Relative Mixing Efficiency

The mixing efficiency is an important parameter of an agitation system. Of course, the decisive factor in the evaluation of the efficiency of a mixer is the degree of homogeneity obtained after a certain time. The mixing efficiency of a particular mixer is relatively high when a minimal power and a short mixing time are required to achieve a specified degree of mixing. In some instances, the mixing efficiency can be extremely low, for example, when whirling and fluid motion are produced only locally without contributing in any way to the mixing within the main body of the fluid.

Since it is impossible to define a perfect mixer, a relative mixing efficiency is defined with respect to the performance of an arbitrary, hypothetical mixer.

In a mixing vessel, the power consumption for laminar flow is given by

$$P = C_1 \mu N^2 d^3 \tag{1}$$

and the mixing time t_m for helical ribbon agitators is inversely proportional to the rotational speed N of the impeller (Coyle et al., 1970; Chavan and Ulbrecht, 1973; Gray, 1963, Hoogendoorn and Den Hartog, 1967; Johnson, 1967; Moo-Young et al., 1972; Nagata et al., 1957):

$$t_m = \frac{C_2}{N} \tag{2}$$

Therefore, the energy input in the vessel is

$$Pt_m = C_1 C_2 \mu N d^3 \tag{3}$$

We define a hypothetical reference mixer by arbitrarily choosing the product C_1C_2 equal to 1 000. This value is chosen from literature data as the lower estimated limit of the product of the power consumption and mixing time.

The efficiency of a given system when compared to our reference mixer is the relative efficiency defined by

$$(\text{eff})_{\text{rel}} \equiv \frac{1\,000 \mu N d^3}{Pt_m} \tag{4}$$

where the product Pt_m is the actual energy required to obtain a given degree of mixing.

Development of a Power Consumption Model

The model is based on the principle that the torque exerted on a rotating impeller is due to the drag force exerted by the fluid flow around the impeller blade. This is in contrast to the models proposed by Bourne and Butler (1965) and Chavan and Ulbrecht (1973*b*), where it is assumed that the power consumed is that resulting from a Couette flow, that is, flow between two coaxial cylinders.

The observed flow patterns that form the basis of the model are illustrated in Figure 2. The horizontal motion of the fluid is qualitatively illustrated in Figures 2*a* and 2*b* for fixed coordinates and relatively to the blades, respectively. The blade velocity is much larger than the fluid velocity.

The forces exerted by the fluid on the blade at a given point are caused by the dynamic pressure of the stream velocity whose action is normal to the blade (the resulting force is generally defined as normal drag or form drag) and the tangential stresses which result in the friction drag or skin drag as the fluid element slides along the surface of the blade and which finally departs owing to its relative motion with respect to the blade. It is evident that the total torque exerted on the impeller is the product of the impeller radius and the sum of the two drag forces over the total impeller blade surface. The torque exerted on the impeller blades is given by

$$\tau = \iint_A - [\mathbf{n}\cdot(\underline{\tau} + p\underline{\delta})]_\theta r dA \tag{5}$$

The term $-[\mathbf{n}\cdot(\underline{\tau} + p\underline{\delta})]_\theta$ is the component of the force per unit area in the θ direction, which is the direction of impeller rotation. However, since r does not vary must for helical ribbon blades, Equation (5) is approximated by

$$\tau = F_{k\theta} R_b \tag{6}$$

and the power is given by

$$P = \tau(2\pi N) = 2\pi R_b N F_{K\theta} \tag{7}$$

The drag force can be expressed in terms of the drag coefficient:

$$F_K = C_D\left(\frac{1}{2}\rho V_\infty^2\right)A \tag{8}$$

Unfortunately, it appears that no data or correlations on the drag flow about an inclined plane are available in the literature. To obtain an approximate expression for the θ component of F_k, it is assumed the total drag is the sum of two drag forces, one for a plate normal to the main flow field and the second for a plate parallel to the flow.

It is further assumed that the approaching fluid velocity is in the θ axis [in Carreau et al. (1976) it was shown that the main component of the flow was tangential]; that is

$$V_\infty \simeq 2\pi N_r R_c = 2\pi(N - N_f)R_c \tag{9}$$

It follows that

$$F_{K\theta} \simeq 2\rho(\pi N_r R_c)^2[C_{Dn}A_n + C_{Dt}A_t] \tag{10}$$

The characteristic area of the blades for the normal flow is appropriately defined by

$$A_n = n_b(l \sin\psi)w \tag{11}$$

For the tangential flow, the characteristic area is not as well defined, since an incoming fluid element slides over the blades in the radial and tangential directions at the same time. We take

$$A_t = n_b(l\cos\psi)\frac{w}{2} \tag{12}$$

assuming that, on the average, the flow is divided in two halves on the front side of the blades and that very little drag results from skin friction on the backside of the inclined blades.

The expressions for the drag coefficient are obtained from Whitaker (1968):

For normal flow

$$C_{Dn} \simeq \frac{5.8}{(Re_b)^{0.44}}, \quad 2 \leq Re_b \leq 20 \tag{13}$$

For tangential flow

$$C_{Dt} \simeq \frac{3.6}{(Re_b)^{0.7}}, \quad 2 \leq Re_b \leq 20 \tag{14}$$

In both cases, the Reynolds number around the blades

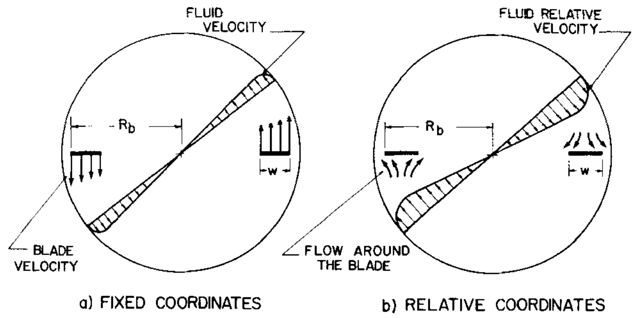

Fig. 2. Tangential velocity profiles.

Re_b is taken to be

$$Re_b = \frac{w 2\pi N_r R_c \rho}{\mu} \tag{15}$$

and the range covered corresponds to the values of our experimental data in the laminar region where $5 < Re_b < 25$. For tangential flow, the assumptions imply that, on the average, the fluid flows tangentially over the blades on a distance equal to w and covers a width equal to $w/2$. From qualitative visual studies of the flow about the blades, these assumptions appear to be reasonable.

From Equations (7), (10), (11), and (12), it follows that the total power is given by

$$P = 4n_b\rho\pi^3 N R_b (N_r R_c)^2 l w [C_{Dn}\sin\psi + \tfrac{1}{2} C_{Dt}\cos\psi] \tag{16}$$

Assuming that $R_c \simeq R_b$ [an assumption already implied in Equation (6)], we obtain the following expression for the power number:

$$N_P \equiv \frac{P}{\rho N^3 d^5} \simeq \tfrac{1}{2}\, n_b \pi^3 \left(\frac{N_r}{N}\right)^2 \left(\frac{lw}{d^2}\right) \left[\frac{5.8\sin\psi\, Re_b^{0.26} + 1.8\sin\psi}{Re_b^{0.7}}\right] \tag{17}$$

In order to calculate the power or the power number, one needs the value of N_r, the relative rotational velocity of the fluid, which can be obtained from experimental data. However, in general, the fluid velocity is much less than that of the impeller, and as a first approximation, we can set N_r equal to N. (This is especially true at low values of N.) Thus, the power can be calculated from the fluid properties, dimensions of the mixing system, and impeller rotational speed. This approximation would predict a too large power consumption for high impeller rotational speeds. A suitable expression for N_r can be developed with the aid of some experimental data.

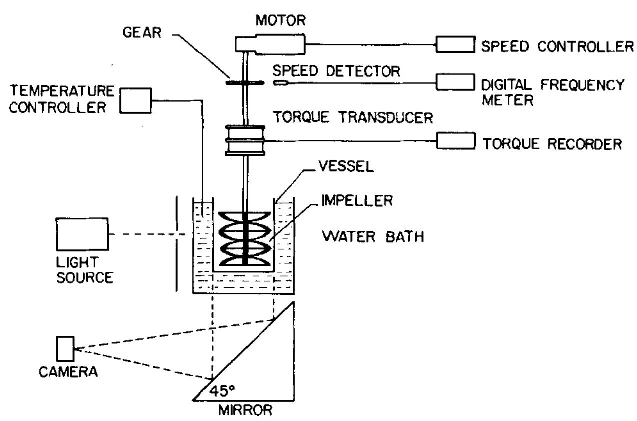

Fig. 3. Experimental apparatus.

TABLE 1. IMPELLERS CHARACTERISTICS

Geometry	Impeller	d (m)	D (m)	h (m)	d_s (mm)	n_b	D/d	l/d	w/d	p/d
A	I	0.130	0.145	0.137	6.35	2	1.11	4.48	0.097	0.719
B	II	0.130	0.145	0.137	6.35	2	1.11	3.00	0.097	1.048
C	III	0.130	0.145	0.137	6.35	2	1.11	4.12	0.195	0.707
D	IV	0.105	0.145	0.137	6.35	2	1.37	4.00	0.121	0.848
E	V	0.130	0.145	0.137	6.35	1	1.11	4.39	0.097	0.695
F	VI	0.222	0.248	0.234	9.53	2	1.11	4.44	0.099	0.690
G	VIII	0.219	0.248	0.238	9.53	2	1.12	4.75	0.072	0.724
H	VI	0.222	0.291	0.234	9.53	2	1.30	4.44	0.099	0.690

TABLE 2. FLUIDS PROPERTIES

Fluid	ρ (kg/m^3)	μ ($N \cdot$ s/m^2)
100% glycerol	1 254	0.568
100% glycerol	1 254	0.708
100% glycerol	1 259	0.800
Silicone oil	1 100	0.137
Vitrea oil	869	0.193

EXPERIMENTAL

The apparatus employed in this study is shown schematically in Figure 3. It consisted of a water bath with the mixing vessel immersed in it. The fluid to be mixed was placed in the cylindrical vessel and the motor-agitator assembly lowered as a unit to place the agitator in the vessel. Three cylindrical mixing vessels were used in this study. The smallest had dimensions of 0.145 m internal diameter and a height of 0.203 m, and the second was 0.248 m internal diameter and 0.305 m high. The largest vessel of 0.291 m internal diameter had a height of 0.38 m.

The agitator was driven by a reversible, variable speed motor coupled to the agitator through a right-angle speed reducer to give minimum and maximum agitator speeds of 0 and 500 rev/min, respectively. The rotational speed was detected by a magnetic transducer, and the torque acting on the agitator was measured by a shaft mounted dynamic torquemeter. The reader is referred to Yap (1976) for a detailed description of the apparatus and methods.

Eight helical ribbon ag'tators constructed from stainless steel were used in this study. They were made by rigidly fixing one or two stainless steel helices (the blades, see Figure 1) to the shaft. In the case of the twin-blade agitators, the helices were mounted 180 deg apart. The details of the impellers' geometries are given in Table 1. Geometries A to E are the combinations of impellers I to V, respectively, with the same 0.145 m vessel. Impeller II has a larger pitch compared to impeller I, while impeller III is wider in blade width and impeller IV has a smaller outside diameter. Impeller V is a single-blade agitator with all dimensions identical to impeller I. Geometry F with impeller VI in the 0.248 m vessel is a scaled-up version of geometry A. Geometry G also uses the 0.248 m vessel, but the agitator has a slightly larger diameter and a smaller blade width compared to combination F. Geometry H is the combination of impeller VI with the larger vessel of 0.291 m diameter.

A tabulation of the fluids used is given in Table 2.

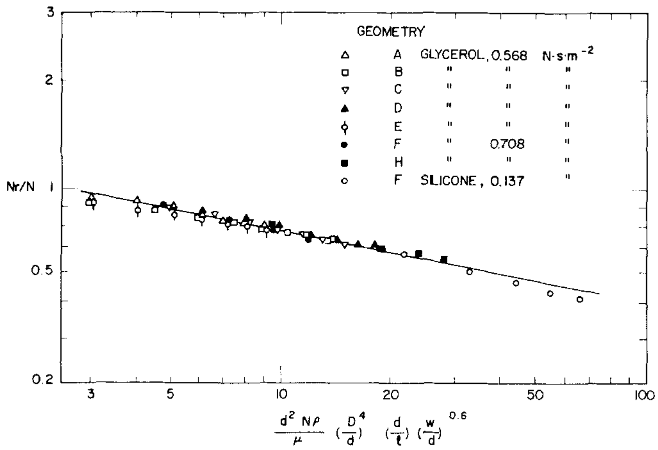

Fig. 4. Correlation of the fluid relative velocity.

RESULTS AND DISCUSSION

The power consumption model requires a determination of the relative fluid-agitator velocities to be useful. The dimensionless analysis of mixing of White and Brenner (1934) suggests the following correlation when the Froude number may be neglected in laminar mixing:

$$\frac{N_r}{N} = a_o\left[\frac{d^2N\rho}{\mu}\right]^{a_1}\left[\frac{D}{d}\right]^{a_2}\left[\frac{l}{d}\right]^{a_3}\left[\frac{w}{d}\right]^{a_4} \quad (18)$$

The five indexes of Equation (18) were determined by a multiple linear regression of the experimental data of the tangential velocity profiles (see Carreau et al., 1976) to give

$$\frac{N_r}{N} = \frac{1}{0.9\left[\frac{d^2N\rho}{\mu}\left(\frac{D}{d}\right)^4\left(\frac{d}{l}\right)\left(\frac{w}{d}\right)^{0.6}\right]^{0.176}} \quad (19)$$

and the fit is shown in Figure 4. The correlation coef-

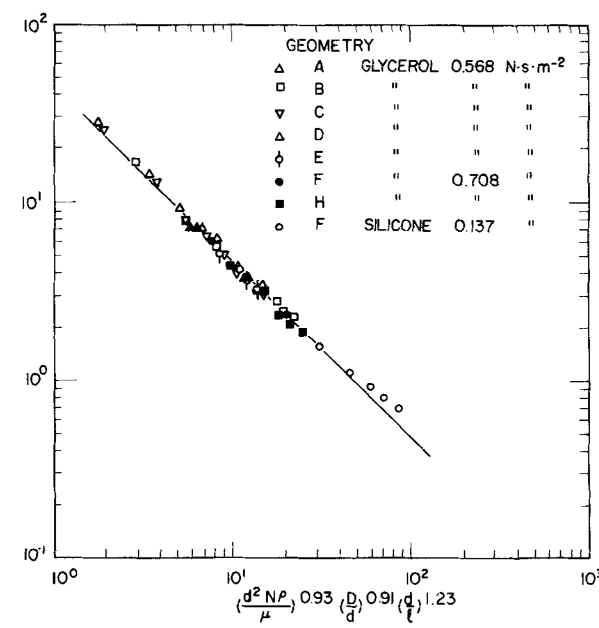

Fig. 5. Generalized correlation of the power number ——— predictions of Equation (21).

ficient is 0.98, with a standard deviation of 0.01. These values are quite acceptable when we consider the scatter of the experimental data. This correlation is obviously restricted to Newtonian fluids and the range of the experimental variables.

Equation (17) proposed for the power consumption can now be solved by substituting the above correlation of the fluid velocity to obtain

$$N_p = 7.9\, n_b \left(\frac{w}{d}\right)^{0.16} \frac{[5.8(Re_b)^{0.26} \sin\psi + 1.8 \cos\psi]}{\left[\left(\frac{d^2N\rho}{\mu}\right)^{0.93}\left(\frac{D}{d}\right)^{0.91}\left(\frac{l}{d}\right)^{1.23}\right]} \tag{20}$$

This equation can be simplified on the following grounds:

1. For commonly used helical ribbon agitators, $w/d \simeq 0.1$, and $\psi \simeq 15$ deg. Therefore, we set $(w/d)^{0.16} \simeq 0.69$, $\sin\psi \simeq 0.258$, and $\cos\psi \simeq 0.965$.
2. Within the range of the experimental conditions, $Re_b^{0.26}$ varies from 1.52 to 2.17. Hence, we take $Re_b^{0.26} \simeq 1.82$ as an average value.

Equation (20) then reduces to

$$N_p = 24\, n_b \left[\left(\frac{d^2N\rho}{\mu}\right)^{0.93}\left(\frac{D}{d}\right)^{0.91}\left(\frac{d}{l}\right)^{1.23}\right]^{-1} \tag{21}$$

Although this result is less accurate than Equation (20), it is much more attractive because of its simplicity. It contains the two most important geometrical ratios: D/d, which expresses the influence of the vessel wall-impeller blade gap, and d/l, which characterizes the geometry of the impeller. The influence of the pitch or the inclination angle of the blades is taken into account through the ratio d/l. The blade width has only a small influence on the power consumption, as indicated by the exponent 0.16 of the ratio w/d. This is surprising, but it can be explained as follows. Although the total blade area submitted to drag action varies linearly with w, the drag coefficients C_{D_t} and C_{D_n} drop as w increases, as shown by Equations (13) and (14). Moreover, the fluid tangential velocity increases slightly with w as shown by the correlation, Equation (19). Thus, the net effect of w on the power consumption is not very important.

The usefulness of Equation (21) was tested using all the power consumption data of the eight geometries with glycerol, silicone, and Vitrea oil. The result is shown in Figure 5; Equation (21), which is the solid line, correlates the data very well with a standard deviation of only 6%. The data for the one-blade impeller (geometry E) have been multiplied by 2, so they could be correlated by the same expression.

Comparison with Literature Data

It is interesting to note that in the power correlation [Equation (21)] the exponent of the Reynolds number is slightly greater than the value of −1 reported by most authors. The value of −0.93, although only 7% different from −1, considerably improves the fit of our data. Correlations of power consumption proposed in the literature to date have been of the form

$$N_p = KRe^{-1} \tag{22}$$

and, in order to test the validity of the present model for the helical ribbon systems used by previous researchers, we force Equation (21) to take the form of Equation (22); that is

Table 3. Comparison of the Proposed Model with Literature Data

Geometry	Reference	d (m)	D/d	h/d	w/d	p/d	H/d	n_b	l/d*	K_{exp}	K_{pred}†	% deviation $\frac{K_{pred} - K_{exp}}{K_{exp}} \times 100$
B-1	Gray (1963)	0.216	1.06	0.941	0.118	0.753	1.15	2	4.1	420	350	16.6
HH-1	Hoogendoorn and Den Hartog (1967)	0.231	1.04	0.94	0.091	0.61	1.56	2	5.1	590	475	19.4
J-1	Johnson (1967)	0.102	1.1	0.966	0.104	0.773	—	2	4.0	300	324	8.0
HC-1	Hall and Godfrey (1970)	0.042	1.11	1.01	0.135	0.517	1.14	1	6.1	230	277	20.4
HC-2	Hall and Godfrey (1970)	0.287	1.10	0.942	0.097	0.495	1.12	1	6.0	207	274	32.3
HC-3	Hall and Godfrey (1970)	0.287	1.10	0.946	0.097	1.00	1.12	2	3.3	250	252	0.8
HC-4	Hall and Godfrey (1970)	0.287	1.11	1.01	0.098	1.00	1.13	1	3.3	130	124	4.6
HC-5	Hall and Godfrey (1970)	0.559	1.10	1.00	0.10	1.00	1.12	2	3.3	246	252	2.4
N-1	Nagata et al. (1956)	0.094	1.06	0.95	0.12	0.74	1.06	2	4.1	328	350	6.7
N-2	Nagata et al. (1956)	0.094	1.06	0.95	0.12	1.11	1.06	2	2.9	267	256	4.1
N-3	Nagata et al. (1956)	0.191	1.05	1.00	0.105	1.00	1.05	2	3.3	300	267	11.0
RB-1	Reher and Bohm (1970)	0.210	1.19	0.952	0.114	1.28	1.19	2	2.5	130	160	23.0
US-1	Ulrich and Schreiber (1967)	0.086	1.07	1.03	0.0875	1.25	—	2	2.8	237	210	11.3
Z-1	Zlokarnik (1967)	0.186	1.02	1.00	0.099	0.50	1.02	2	6.4	1 000	662	33.8

* Calculated.
† $K_{pred} = 30\, n_b\, (d/D)^{1.2}\, (l/d)^{1.3}$.

TABLE 4. RELATIVE EFFICIENCY OF VARIOUS IMPELLERS

(Glycerol, $\mu = 0.568\ N \cdot s/m^2$)

Impeller	n_b	D/d	l/d	w/d	C_2*	(Efficiency)$_{rel}$
I	2	1.11	4.48	0.097	45	0.055
II	2	1.11	3.00	0.097	49	0.100
III	2	1.11	4.12	0.195	22	0.137
IV	2	1.37	4.00	0.121	53	0.086
V	1	1.11	4.39	0.097	58	0.094

* Values obtained from Carreau et al. (1976).

$$N_p = C_3\left[\left(\frac{d}{D}\right)^{1.2}\left(\frac{l}{d}\right)^{1.3}\right] Re^{-1} \quad (23)$$

By curve fitting the data of Figure 5, the value of C_3 was found to be equal to 60 for twin-blade impellers; hence

$$N_p = 30\, n_b \left(\frac{d}{D}\right)^{1.2}\left(\frac{l}{d}\right)^{1.3} Re^{-1} \quad (24)$$

A comparison of the values of K predicted by Equation (24) with the experimental values reported in the literature for a variety of helical ribbon agitators with Newtonian fluids is presented in Table 3. Generally, the percentage deviation between the predicted and experimental values is very acceptable when we consider the experimental errors and the imprecision of l, the length of the blades. It was necessary to calculate l from the pitch and the diameter of the blades, since the factor l/d is not reported in the literature. Nevertheless, almost half of the predicted values are within ±8% of the experimental values. It should be noted that the larger deviations are observed for geometries HG-1, HG-2, RB-1, and Z-1 which had the impeller pitches considerably different from those commonly used. The correlation could be improved for these geometries by correcting for the inclination angle. Table 3 highlights the observation that geometry has a considerable influence on the power consumption. An example is geometry Z-1 which has a value of K eight times larger than that of geometries HG-4 and RB-1. The constant K, however, is independent of the fluid properties for Newtonian fluids.

A few tests were made with the agitator rotating in the clockwise direction (impeller pumping upwards at the vessel wall). The power consumed was found to be about 10% higher than the identical situation with counterclockwise rotation. It is believed that depending on the direction of the agitator rotation, the inertia forces affect differently the flow patterns; hence, Equation (19) should be slightly modified to account for that effect.

Relative Efficiency of Various Agitators

From the known mixing time (Carreau et al., 1976) and the power consumption correlation, we can estimate and compare the relative efficiency of each system in the 0.154 m vessel.

The concept of relative efficiency defined above is used for the calculation. From Equations (2), (4), and (24)

$$(\text{eff})_{rel} = \frac{1\,000}{30\, n_b \left(\frac{d}{D}\right)^{1.2}\left(\frac{l}{d}\right)^{1.3} C_2} \quad (25)$$

The relative efficiencies of the five impellers for glycerol are tabulated in Table 4.

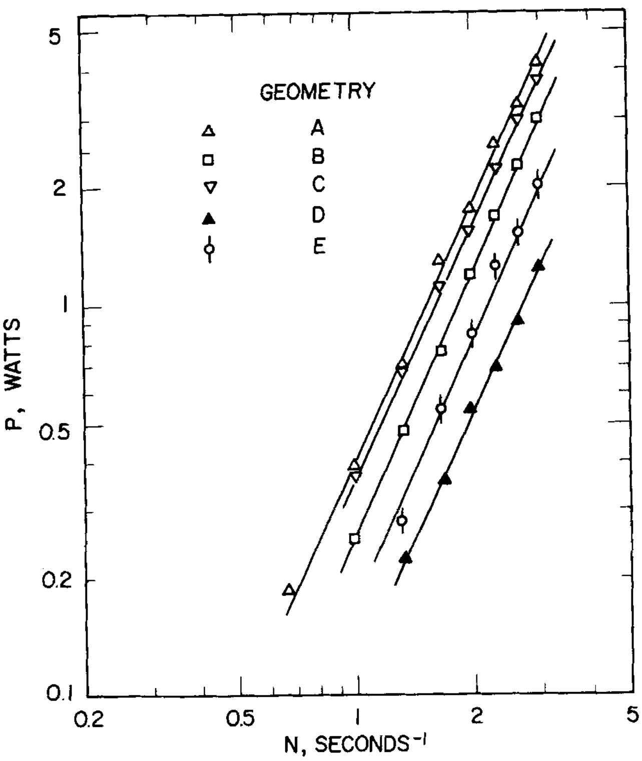

Fig. 6. Influence of geometry on power consumption for glycerol ($\mu = 0.568$ N.s/m^2).

The higher the relative efficiency, the less total energy is required to achieve a certain degree of mixing in a given volume of fluid. Table 4 indicates that impeller III, which has wider blades (w is twice of that of the others), is the most efficient impeller; its efficiency is 2.5 times that of impeller I. A larger blade width results in a higher pumping capacity, but, as discussed earlier, the torque is unchanged. The relative efficiency of impeller II is rather good as it consumes low power, since the ratio l/d is rather small (high pitch of the blades) compared with the others. The efficiency of impeller IV is better than that of impeller I; because of the larger gap between the vessel wall and the blades (23% larger), the power consumed is considerably less. The one-blade impeller V has only a slightly lower pumping capacity or lower mixing effectiveness; however, since its power consumption is approximately half of that consumed by impeller I, its efficiency is almost doubled.

Scale-up and Design Aspects

Since it is neither convenient nor economical to experiment with industrial size units to get optimal process conditions, it is desirable to have a scale-up technique to predict the performance of an industrial scale mixer from experiments carried out on laboratory size equipment.

In this section, the scale-up possibility of the systems studied is discussed. The implication and extrapolation of the variables studied and their effects are briefly reviewed. To facilitate the discussion that follows, power consumption data are plotted in a noncorrelated form in Figure 6, showing the effects of the geometrical variables for glycerol.

To predict the scale-up performance of agitators in production size mixing systems, information about the quality or degree of mixing and energy consumed should be compared. Dimensionless correlations available can be used to conveniently scale-up geometrically similar systems. In the so-called laminar mixing with helical ribbon agitators, Nt_m, which is the number of impeller revolutions needed to obtain an arbitrary but constant degree of homogeneity, is constant for a given geometry [Equation (2)].

Thus, for geometrically similar systems, the impeller rotational speed to maintain a given mixing time is independent of the scale of the system. The power number is inversely proportional to the Reynolds number, as expressed by Equations (22), (23), or (24). The combination of Equations (2) and (24) gives

$$\frac{t_m{}^2 P}{d^3 \mu} = C_2 K \tag{26}$$

where for Newtonian fluids C_2 and K are functions only of the geometrical ratios. For a given mixer geometry, $d/D = C_3$, a constant; hence

$$\frac{t_m{}^2 P}{D^3 \mu} = C_2 C_3 K = \text{constant} \tag{27}$$

Therefore, the power per unit volume required to obtain a certain homogeneity is independent of the vessel size and is proportional to the fluid viscosity.

Effects of the Geometrical Ratio D/d and Tank Diameter

The effect of the gap between the vessel wall and the impeller blades is accounted for by the geometrical ratio D/d. The effect of D/d in the range $1.11 \leq D/d \leq 1.37$ (this work) is predicted accurately by the proposed equation. The predictions for values of $1.01 \leq D/d \leq 1.11$ (literature data, Table 3) vary from good to accurate. Mixing systems with a scale-up factor of 5 (that is, 0.76 m diameter vessels) can well be described by the results of this work. The effectiveness obtained for our 0.154 m and 0.254 m vessels is comparable to that obtained by Coyle et al. (1970) in a 0.76 m tank, with $D/d \simeq 1.03$; the power predicted by Equation (24) for the 0.559 m vessel of Hall and Godfrey (1970) is within 2.4% of the data. The performance of large scale mixing systems is predictable for geometrical ratios in the range of the values studied in this work.

Effect of the Pitch Ratio

Pitch ratios ranging from 0.6 to 1.04 have been tested. For smaller pitch ratios, we do not expect the drag model to hold for blade inclination angles smaller than 15 deg, and the blades are almost tangent to the vector of the oncoming fluid velocity. Then skin friction dominates on both sides of the blades, resulting in an overall drag larger than accounted for by the simple model of Equation (24). As expected, the power predicted for Zlokarnik's system, in which case the impeller had a pitch ratio of 0.5, is too low (see Table 3). The power consumption also increases considerably as the pitch ratio decreases owing to the interrelation between the pitch and the blade length (see Table 1). Finally, impellers with pitch ratios equal to 0.5 or smaller would have a low pumping capacity or mixing rate, resulting in a very poor efficiency.

Effect of the Blade Width

The blade width-to-diameter ratio varied from 0.09 to 0.19 in this work. Doubling the blade width doubles the pumping capacity and the mixing rate. However, as discussed, the power consumed is almost independent of the blade width. The assumptions leading to the proposed model become less valid as the ratio w/d is increased. Good predictions are obtained for values of $w/d \leq 0.2$, but additional tests are required to confirm the model's adequacy beyond this point. Although impellers with wider blades may appear interesting from the point of view of efficiency, they are difficult to build and hence expensive.

Effect of the Number of Blades

For the single-blade helical ribbon impeller, the mixing time is about 20% longer than with the similar twin-blade impeller, at the same rotational speed. However, the power consumed for the one-blade impeller is approximately half of that consumed by the two-blade impeller. Its efficiency is almost double, and hence one-blade impellers could be quite interesting for design purposes. More experimentation on one-blade impellers is required in order to draw definite conclusions.

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NOTATION

a_0, a_1, a_2, a_3, a_4 = indexes in Equation (18), dimensionless
A = impeller blade area, m^2
A_n = impeller blade area normal to the flow, m^2
A_t = impeller blade area tangential to the flow, m^2
$C_1 C_2$ and C_3 = proportionality constants, dimensionless
C_D = total drag coefficient, dimensionless
C_{Dn} = normal drag coefficient, dimensionless
C_{Dt} = tangential drag coefficient, dimensionless
d = diameter of impeller, m
d_s = diameter of impeller shaft, mm
D = diameter of vessel, m
F_K = drag force, N
$F_{K\theta}$ = drag force in the θ direction, N
h = height of impeller, m
H = height of liquid in vessel, m
K = proportionality constant defined by Equation (22), dimensionless
l = length of impeller blade, m
$\mathbf{n}$ = unit vector normal to the blade surface
n_b = number of blades
N = rotational speed of impeller, rev/s
N_f = rotational velocity of the fluid, rev/s
N_P = power number, $P/\rho N^3 d^5$, dimensionless
N_r = rotational velocity of the fluid relative to the impeller, rev/s
p = impeller pitch, m; or fluid pressure in Equation (5), N/m^2
P = power consumed, W
r = radial position, m
R_b = radius of impeller blade, m
R_c = radius of the center of impeller blade, m
Re = Reynolds number for mixing systems, $d^2 N \rho/\mu$, dimensionless
Re_b = Reynolds number about the blade, $w 2\pi N_r R_c \rho/\mu$, dimensionless
t_m = mixing time, s
V_∞ = approaching fluid velocity, m/s
w = blade width, m

Greek Letters

$\underline{\underline{\delta}}$ = unit tensor
μ = fluid viscosity, $N \cdot s/m^2$
ρ = fluid density, kg/m^3
$\underline{\underline{\tau}}$ = stress tensor, N/m^2
ψ = blade inclination angle, deg
τ = torque, $N \cdot m$

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Part III. Non-Newtonian Fluids

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The use of helical ribbon agitators to mix viscous non-Newtonian fluids has been investigated. A generalized model, based on an appropriate definition of effective viscosity, is proposed to predict power consumption. This model is most successful with fluids that do not have a high degree of elasticity.

It was found that the efficiency of mixing of pseudoplastic fluids was about half of that of Newtonian fluids in the same mixer, while the efficiency of mixing viscoelastic fluids was still lower and approximately independent of the mixer geometry. Blade width was the primary variable affecting the mixing efficiency on inelastic fluids.

SCOPE

The mixing of very viscous fluids can be efficiently accomplished with a helical ribbon agitator (HRA). A model to predict the power consumption of the HRA mixing Newtonian fluids has been reported by Patterson et al. (1979); however, very viscous fluids are often non-Newtonian or viscoelastic in nature. This paper reports the experimental results of various HRA's mixing pseudoplastic and viscoelastic fluids. Six different mixer geometries with three different fluids were examined. The fluids varied from negligible elasticity (2% aqueous Natrosol) to high elasticity (1% aqueous Separan). The model previously derived has been generalized to include the mixing of non-Newtonian fluids. This was accomplished by defining an effective rate of deformation that is functionally dependent on the agitator geometry rotational speed and

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